Contouring accuracy,
Jerk, productivity,
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CONTOURING ACCURACY OF A MACHINE TOOL: DESIGN OF A PERFORMANCE TEST AND OPTIMIZATION OF THE JERK.

Abstract - With increasing demand of high feed rates the dynamical characterization of a milling center is becoming a strategic aspect. The contouring accuracy at high feed rates and high acceleration is vital in order to preserve the tolerance integrity of parts produced with high speed machining. The dynamic accuracy is influenced by the velocity, acceleration and Jerk. While the cutting speed depends from technological considerations and the maximum velocity and the acceleration depends from the mechanical structure, what value assign to the Jerk is not well defined. The Jerk has an important impact on the execution time of a tool path in a mould/die production, where there are frequent accelerations and decelerations, and a high jerk leads to a deteriorated surface accuracy and an unsmooth machining process. In this paper, experimental test were conducted on various milling centers in order to define the Jerk value. Firstly, some tool path features are introduced in order to consider the effect of the trajectory. Then a tool path, called STAR, is designed and it has been tested by changing the Jerk. A mathematical model able to estimate the execution time it was prepared. A performance test is designed in order to estimate the contouring accuracy in relation with Jerk for all tool paths. The best value of Jerk is the compromise between high accuracy and high productivity. Then an objective function is introduced in order to optimize the Jerk.

1. INTRODUCTION

Improvement of machine tool accuracy is an essential part of quality control in manufacturing processes. There is constant pressure on industrial manufacturers to produce high-quality products while maintaining high productivity.

Errors in the final dimensions of the machined part are determined by the accuracy with which the commanded tool trajectory is followed, combined with any deflection of the tool, part/fixture, or machine caused by the cutting forces.

Machine tool performance and consistency is the main determinant of the quality of parts machined by it. It is of importance to check the performance of the machine tool systematically for direct quality control purposes or to compensate for this uncertainty. Schlesinger [13] was the first to provide a systematic testing method for machine tools. Some authors have used the grid encoder in order to perform free-form 2D contouring test. Jywe [7] has designed a cheaper instrument to perform it. Flores [6] evaluates the execution time and the dynamic accuracy when a linear interpolation or circular has been performed.

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The Jerk (rate of change of velocity) has an important role in the accuracy of the machined part, especially for high-speed machining of complex surface. Several authors have studied the feedrate scheduling considering Jerk limitation (Erkorkmazk [5], Altintas [1], Dong [3]). It was not found articles about the determination of the Jerk and how determine it.

It is known that machined part accuracy has four major contributors: geometric errors of the machine construction, thermally induced errors from heat sources associated with the machine/cutting process, trajectory following errors caused by controller and machine structural dynamics, and errors due to the cutting forces. The grid encoder highlights geometric errors of the machine construction and trajectory following errors caused by controller and machine structural dynamics. This can be considered a good approximation of the total error for finishing operation, where the cutting forces are low.

In Schmitz [12] a case study, performed on a new high-speed machining center is described, where the contribution of the geometric, thermal, controller, and cutting forces errors to inaccuracies in geometry for the well-known circle-diamond-square test part have been quantified. The trajectory is recorded by the grid encoder.

Using current technology, it is possible to measure the quasi-static geometric errors of a machine tool and their thermally induced deviations. With these data, it can construct a thermal/geometric error model of the machine that predicts the static tool point positioning error anywhere in the workspace and at any thermal state (Donmez [4]). This machine error model can be used to predict tool position errors at discrete points along arbitrary CNC paths and predict the dimensional errors of a machined part that are caused by the imperfect machine geometry. It is also possible to machine parts using the same CNC part paths and then measure the actual dimensions of the resulting parts. Comparisons of the predicted and actual part dimensions show that the thermal/geometric error model obtained from static measurements is capable of predicting some, but not all, of the resulting part dimensional errors (Donmez [4]). To improve the accuracy of models seeking to predict part dimensions, it is necessary to extend these models to include trajectory following errors related to the controller.

Srinivasa [14] studied the spindle thermal drift, which is believed to be the dominant source of errors among thermally induced errors. In the paper a laser ball bar (LBB) was used.

Schmitz [9] demonstrates the use of an instrument capable of measuring arbitrary, dynamic tool paths through three-dimensional (3-D) space with micrometer-level accuracy.

The surface location errors, or workpiece geometric inaccuracies that result from dynamic displacement of the tool and/or workpiece during stable machining was studied from several authors ([11], [10] and [8]).

In this paper, a performance test based on the grid encoder of Heidenhain has been designed. The experiments are carried out on two milling center, a new one and the other one in work. Next, the Jerk was optimized considering the productivity and the contouring accuracy. The productivity has been estimated through a simple mathematical model and the contouring accuracy derives from the performance tests.

2. EXPERIMENTAL SETUP

The tests have been performed on two milling centres:

1- Four axis milling centres (built by MANDELLI Company, located in Piacenza), said “SPARK”. Maximum acceleration $a_{max} = 4$ *m/s*^{2}, maximum ranges of X-axis 1600 *mm*, Y-axis 1400 *mm*, Z-axis 1400 *mm*. The chief technical features include a turning table with a 70 kW torque motor and a speed of up to 600 rpm.

2- Five axis-milling centres, said JOTECH (built by JOBS Company, located in Piacenza). The ranges are X-axis 1600 *mm*, Y-axis 800 *mm*, Z-axis 600 *mm*, equipped with linear motors on the X and Y axis. Maximum acceleration $a_{max} = 3$ *m/s*^{2}.

The movement of the milling head is recorded by the grid encoder KGM 182 of HEIDENHAIN (shown in the Fig. 1).

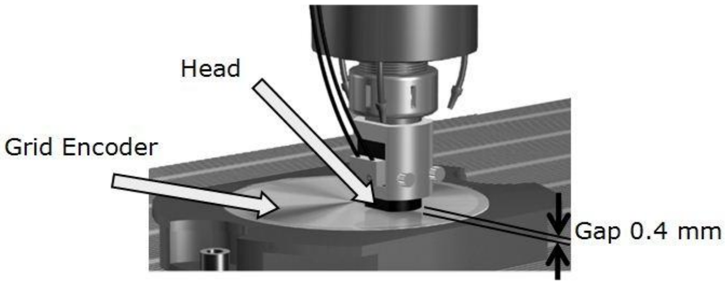

Fig. 1. Components of the grid encoder HEIDENHAIN KGM182

The design of the experiment is shown in the Tab. 1.

Tab. 1. Design of experiment

	SPARK 1600	**JOTECH**
Velocity (m/min)	1, 5, 10	10, 20
Jerk (*m/s*^{3})	25; 50	SRAMPTIME (0, 50, 100)
Numerical Control	Sinumerik 840D	FIDIA C2
Tool paths	STAR, SQUARE	CORNER

3. DESIGN OF THE TRAJECTORY “STAR”

The tool paths in milling machining may have, basically, two types of geometric singularities (Fig. 2): the first type in the corner when the milling head stops due to the inertial forces, mainly influenced from the discontinuity of the velocity profile, and the second type during the execution of a circle because of centrifugal force.

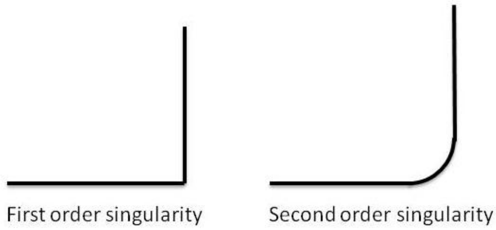

Fig. 2. Types of tool paths singularities

A new tool path, said STAR, has been designed, in order to perform a single test. The dimensions and the direction of the movement are shown in the Fig. 3.

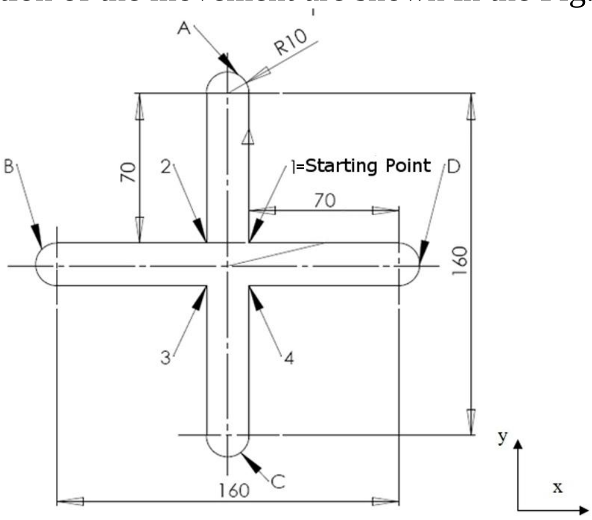

Fig. 3. Characteristics of the STAR trajectory (dimensions in mm)

The square and the corner have the dimensions shown in the Fig. 4.

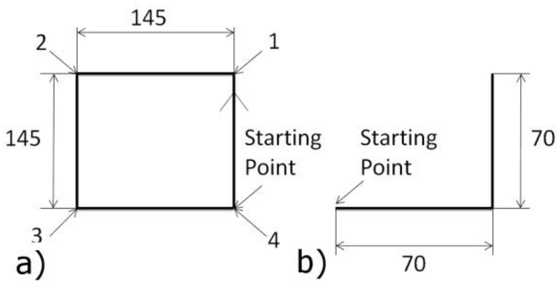

Fig. 4. Characteristics of the SQUARE trajectory a) and CORNER b) (dimensions in mm).

4. MODELLING

In this chapter a model, able to predict the execution time, will be presented. The model involves the effect of the Jerk. The scheme of optimization is also presented.

3.1 TIME PREDICTION

The velocity profile is shown in the Fig. 5. It consists of the limited jerk *J* section (*0<t<tA*, *tB<t<t1*, *t1+t2<t<tC* and *tD<t<tend*), constant acceleration *a* section (*tA<t<tB* and *tC<t<tD*) and the constant velocity *V* section (*t1<t1+t2*). At t_{end} the milling head has reached the final point of the tool path at the distance *S* from the origin point.

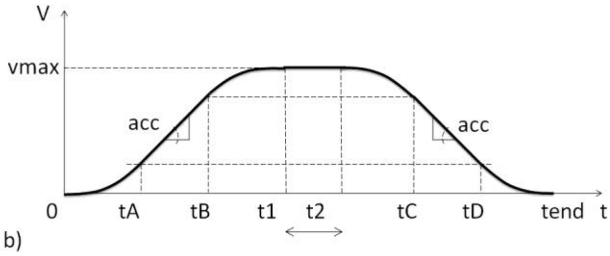

Fig. 5. Velocity profile

The stretch with constant velocity is said t_2. It can be shown that:

$$t_2 = S/V - a/J - V/a$$

The total time to traverse the distance *S* is:

$$t_{tot} = S/V + a/J + V/a$$

The Jerk influences the velocity profile, as shown in Fig. 6.

It can be observed, that exists a Jerk limit value below which the acceleration has to be decreased in order to match the boundary condition (Fig. 6 *d*). The limit is $J_{\lim} = a^2/V$. The acceleration is adjusted to the value $a_{adj} = \sqrt{V \cdot J}$

Finally the total time is:

$$\text{If } J \geq a^2/V \text{ then } t_{tot} = S/V + a/J + V/a.$$

Else

$$a_{adj} = \sqrt{V \cdot J} \text{ then } t_{tot} = S/V + 2 \cdot \sqrt{V}/\sqrt{J}.$$

For the STAR trajectory the total time is: $t_{STAR} = 4 \cdot t_{tot} + 4 \cdot t_{circ}$. The circular section is run with a constant velocity. For the square: $t_{square} = 4 \cdot t_{tot}$, and for the corner: $t_{corner} = 2 \cdot t_{tot}$.

The JOTECH is equipped with FIDIA control numeric. The Jerk can be changed with the parameter SRAMPTIME. The relation between SRAMPTIME and the Jerk is:

$$J = a/(TRAMP/2).$$

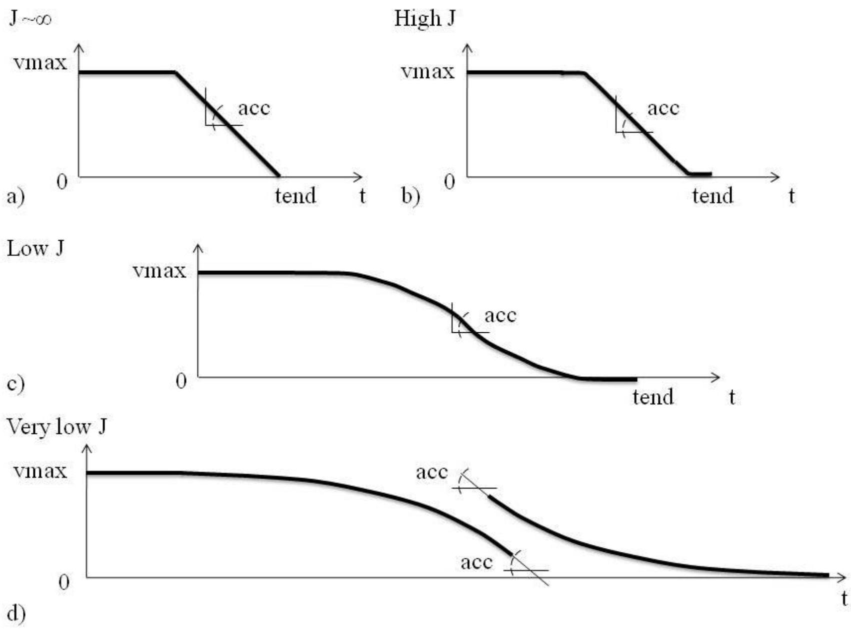

Fig. 6. Velocity profile with different values of Jerk (J).

3.2 OPTIMIZATION STRATEGY

The optimization strategy can be summarized in these steps:

1- **Definition of a cost function.** The cost function increases with the tool path error and the execution time. Considering the ranges of the error and the execution time, the relation between the cost function and the contouring accuracy and productivity is assumed linear:
$Cost = k_1 \cdot Error + k_2 \cdot Time$, with k_1 and k_2 > 0. k_1 and k_2 are obtained from decision of the production manager. The question is: how much the production manager wants to decrease the error and increase the execution time?
Roughly the situations can be:
  1- The time is important, then $k_2 >> k_1$.
  2- The error is also important, then $k_2 > k_1$.
  3- If the actual situation is perceived as the best (for existing machines), the ratio k_2/k_1 can be estimated and then analyze the effect of the feedrate and acceleration.

2- **The function of the error and productivity respect to the Jerk.** The function of the error respect to the Jerk is experimentally determined. The productivity is estimated by the model.
Demaurex [2] has demonstrated that for one DOF mechanical system, without damping and not controlled, the relation between Jerk (because the Jerk profile has discontinuities) and the overshoot is linear. It is pointed out that the deviation measured by the grid encoder is the result of dynamic errors and the controller, so the linear relation of Demaurex [2] should be deepened, but it is not the goal of this work. In this paper, it is assumed that $Err = D \cdot J + E$, where D and E are determined experimentally.

The total time is:

$$\begin{cases} t_{tot} = S/V + a/J + V/a, & if \quad J \geq a^2/V \\ t_{tot} = S/V + 2 \cdot \sqrt{V}/\sqrt{J}, & if \quad J \leq a^2/V \end{cases}$$

Finally, $Cost(J) = k_1 \cdot D \cdot J + k_1 \cdot E + k_2 \cdot t_{tot}$. If the Jerk increases then the error increases, and the time decreases.

3- **Optimization of the function.** In general it can be written:
$dCost/dJ\,(J) = k_1 \cdot dErr(J)/dJ + k_2 \cdot dT(J)/dJ = 0$, with $dT(J)/dJ < 0$; the second derivative is:
$d^2Cost/dJ^2\,(J) = k_1 \cdot d^2Err(J)/dJ^2 + k_2 \cdot d^2T(J)/dJ^2 = k_2 \cdot d^2T(J)/dJ^2$. The minimum exists if and only if $d^2T(J)/dJ^2 > 0$. Substituting the error and the productivity:

$$\begin{cases} dCost/dJ\,(J) = k_1 \cdot D - k_2 \cdot a/J^2, & if \quad J \geq a^2/V \\ dCost/dJ\,(J) = k_1 \cdot D - k_2 \cdot \sqrt{V}/J^{3/2}, & if \quad J \leq a^2/V \end{cases}$$

The optimum value is :

$$\begin{cases} J_{opt} = \sqrt{(k_2/k_1) \cdot (a/D)}, & if \quad J \geq a^2/V \\ J_{opt} = \left((k_2/k_1) \cdot \left(\sqrt{V}/D\right)\right)^{2/3} & if \quad J \leq a^2/V \end{cases}.$$

3 RESULTS

In this chapter, the results of the contouring accuracy for all the trajectories will be summarized. Next, the results of the optimization are presented and discussed.

5.1 CONTOURING ACCURACY

In this section results, referred to the contouring accuracy, are presented.

In Fig. 7 the real tool path is presented together to the maximum overshoot. The aim is to determine the value of the sensitivity D in $\mu m/(m/s^3)$, and analyze the influence of the feedrate and the tool paths.

For SPARK, the results are shown in the Fig. 8, Fig. 9 and Fig. 10.

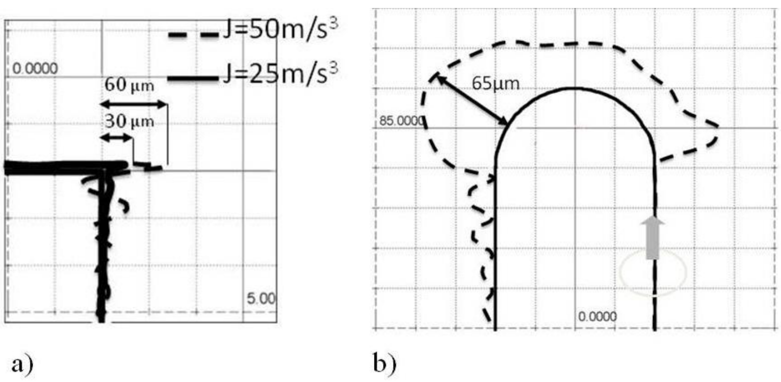

Fig. 7. Recording of the STAR by the grid encoder in 2 a) and in A b) for the velocity 5 m/min

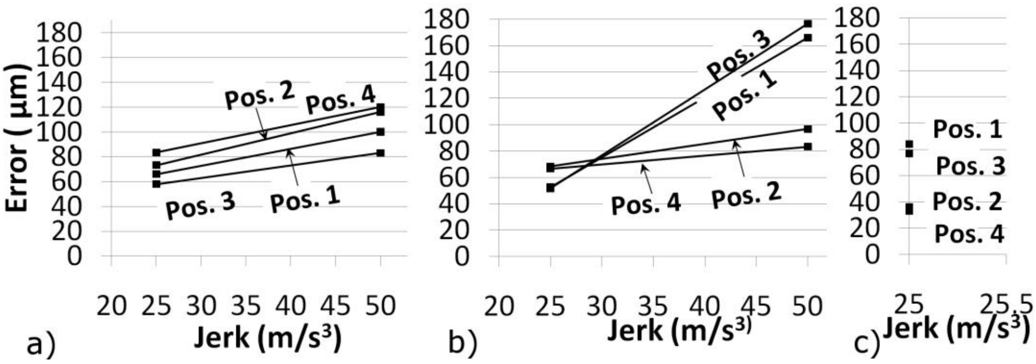

Fig. 8. Maximum overshoot during the STAR a) 10 m/min, b) 5 m/min and c) 1m/min (SPARK)

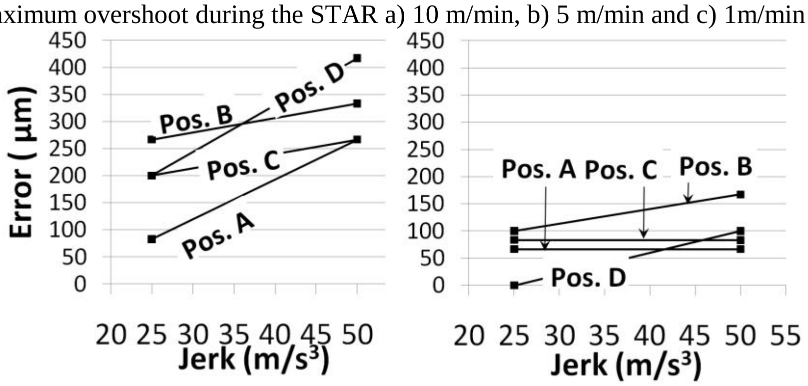

Fig. 9. Maximum circular error during the STAR a) 10 m/min, b) 5 m/min (SPARK)

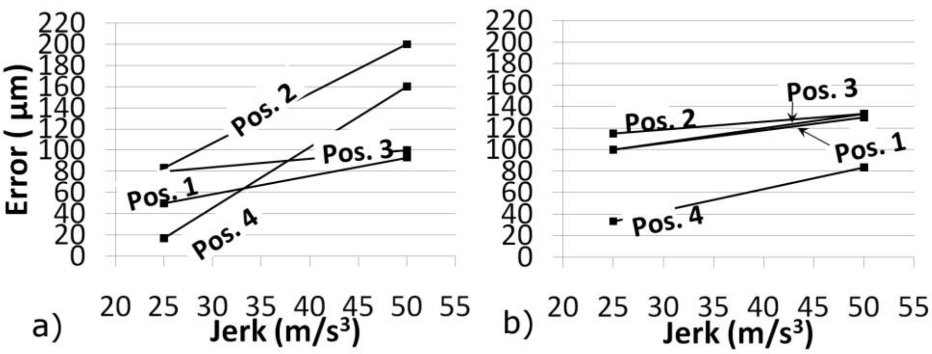

Fig. 10. Maximum overshoot during the SQUARE a) 5 m/min, b) 10 m/min (SPARK)

It can be noted that the velocity and the position influences the absolute value of the overshoot. Regard to the *D*, the effect of the velocity is different. The Tab. 2 shows the results of the *D*, in terms of mean values between the positions characterized from the same axis movement.

Tab. 2. Results of *D* in $\mu m / \left(m/s^3 \right)$

	POS. 1 (A) & 3 (C)		**POS. 2 (B) & 4 (D)**	
	5 m/min	10 m/min	5 m/min	10 m/min
STAR	4.7	1.05	1.2	1.55
SQUARE	1.2	1.2	5	1.5
CIRCLE in the STAR	2	2.7	1.32	5.6

D is high for the position 1 and 3, regard to the STAR, and 2 and 4 for the SQUARE, with feed-rate of 5 *m/min*. This demonstrates that the movement along X-axis is critical.

The error for the JOTECH, when the corner is executed, is shown in Fig. 11.

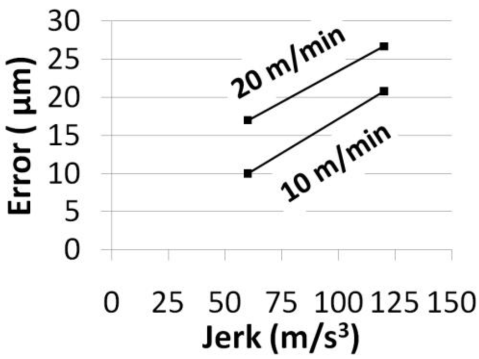

Fig. 11. Maximum overshoot error during the CORNER, varying the feedrate (JOTECH)

The $D = 0.17\ \mu m/\, m/s^3$. Some tests, performed rotating the corner, show that the position is not significant.

5.2 OPTIMIZATION RESULTS

The SPARK was a new machine tool when the test were performed. Since the ratio k_2/k_1 is not given, the aim of the optimization analysis is to estimate the influence of the k_2/k_1 on the optimal value of the Jerk, in order to be a support for the user of the machine. The optimal value is plotted as a function of the ratio k_2/k_1 in Fig. 12. In order to determine the optimal values of Jerk, two values of D have been selected, the minimum and the maximum values derived from the previous analysis. The feedrate is 10 *m/min*.

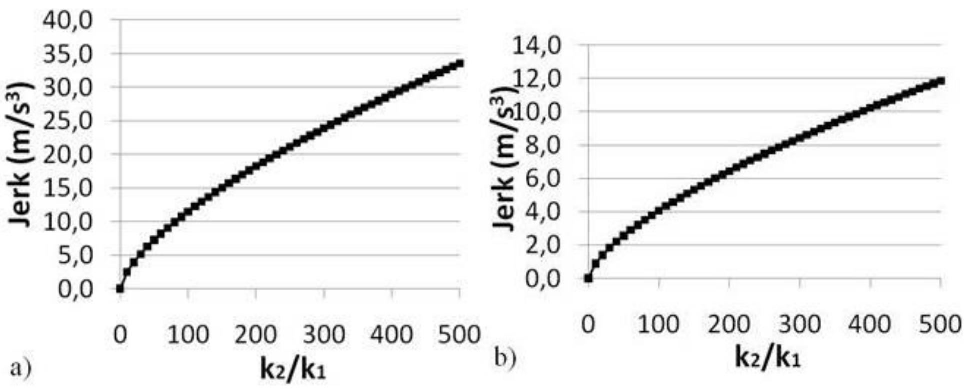

Fig. 12. Plot of the optimal Jerk vs. the ratio k_2/k_1, with D = 1.05 a) with D = 5 b) (SPARK)

The production manager can determine the optimal value of Jerk referred to a value of ratio k_2/k_1.

The JOTECH is an existing milling center and the production is perceived as satisfactory. Based on this information the ratio k_2/k_1 can be estimated. In this case, the aim of the analysis is to estimate the optimal Jerk varying the feedrate.

The actual situation is S*RAMPTIME*=100 *ms*, that corresponds to the Jerk = 60 m/s^3. The normal feedrate is 10 *m/min*. Then the ratio k_2/k_1 can be estimated:

with D = 0.17 $\mu m/\,m/s^3$, $k_2/k_1 = J^2 \cdot D/(a) = 200\,\mu m/s$ (because $J \geq a^2/V$).

If S*RAMPTIME* is the same but the velocity changes, if the $V \leq a^2/J \leq 0.15 m/s = 9 m/\min$ then the relation between the optimal Jerk and the feed-rate is shown in the Fig. 13.

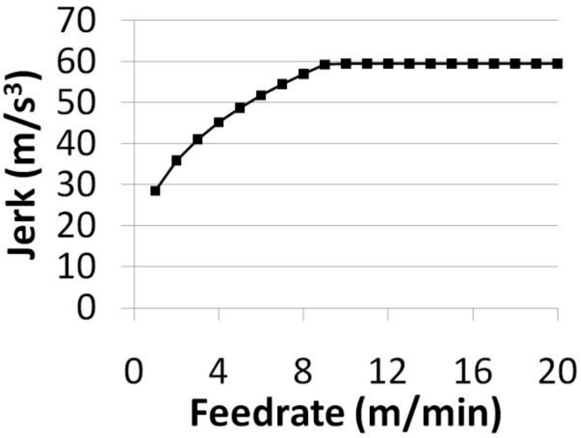

Fig. 13. Plot of the optimal value of the Jerk varying the feedrate (JOTECH).

5.3 REMARKS

The optimal value of Jerk for two milling centres have been determined. This is the best compromise between the contouring accuracy and productivity. This section would answer the question: how much is the cost saving?

For SPARK: The k_1=1 $U.C./\mu m$ and k_2=100 $U.C./s$ (the U.C. indicates the unit of cost). With the feedrate of 10 *m/min* and D = 1.05 $\mu m/\,m/s^3$, the optimal Jerk is 11.5 m/s^3 (Fig. 12). It is assumed that E = 46.8 μm and S = 171.4 *mm*. If the Jerk remains 25 m/s^3 :

$$C_{opt}\left(11.5\ m/s^3\right) = k_1 \cdot D \cdot J_{opt} + k_1 \cdot E + k_2 \cdot \left(S/V + 2 \cdot \sqrt{V}\Big/\sqrt{J_{opt}}\right) = 185.81 U.C.$$

$$C_1\left(25\, m/s^3\right) = k_1 \cdot D \cdot J_1 + k_1 \cdot E + k_2 \cdot \left(S/V + 2 \cdot \sqrt{V}\Big/\sqrt{J_1}\right) = 192.18 U.C.$$

And the cost saving is $\left(C_1 - C_{opt}\right)/C_1 \cdot 100 = 3.31\%$

If D = 5 $\mu m/\,m/s^3$ and E = 73 μm then the optimal Jerk is 4.1 m/s^3 (Fig. 12):

$$C_{opt}\left(4.1\ m/s^3\right) = k_1 \cdot D \cdot J_{opt} + k_1 \cdot E + k_2 \cdot \left(S/V + 2 \cdot \sqrt{V}\Big/\sqrt{J_{opt}}\right) = 236.68 U.C.$$

$$C_1\left(25\, m/s^3\right) = k_1 \cdot D \cdot J_1 + k_1 \cdot E + k_2 \cdot \left(S/V + 2 \cdot \sqrt{V}\Big/\sqrt{J_1}\right) = 317.18.C.$$

And the cost saving is $\left(C_1 - C_{opt}\right)/C_1 \cdot 100 = 25.3\%$

For the JOTECH: The feedrate is 2 *m/min*, and then the optimal value is 36 m/s^3 (from Fig. 13). k_1=1 $U.C./\mu m$ and k_2= 200 $U.C./s$. $E \approx 0$ and S=70 mm. If the Jerk remains 60 m/s^3:

$$C_1\left(60\, m/s^3\right) = k_1 \cdot D \cdot J_1 + k_2 \cdot \left(S/V + 2 \cdot \sqrt{V}\big/\sqrt{J_1}\right) = 439.6 U.C.$$

$$C_{1-opt}\left(36\, m/s^3\right) = k_1 \cdot D \cdot J_{1-opt} + k_2 \cdot \left(S/V + 2 \cdot \sqrt{V}\big/\sqrt{J_{1-opt}}\right) = 438.3 U.C.$$

And the cost saving would be $\left(C_1 - C_{1-opt}\right)/C_1 \cdot 100 = 0.3\%$.

If the machine tool is deteriorating, D would become 1 $\mu m/\, m/s^3$, and the optimal value of Jerk is $11\, m/s^3$. Then:

$$C_1\left(60\, m/s^3\right) = k_1 \cdot D \cdot J_1 + k_2 \cdot \left(S/V + 2 \cdot \sqrt{V}\big/\sqrt{J_1}\right) = 489.4 U.C.$$

$$C_{1-opt}\left(11\, m/s^3\right) = k_1 \cdot D \cdot J_{1-opt} + k_2 \cdot \left(S/V + 2 \cdot \sqrt{V}\big/\sqrt{J_{1-opt}}\right) = 453 U.C.$$

And the cost saving is $\left(C_1 - C_{1-opt}\right)/C_1 \cdot 100 = 7.43\%$.

6 CONCLUSIONS AND FUTURE DEVELOPMENTS

In this paper a performance test able to define the Jerk value, as the best compromise between the contouring accuracy and the productivity, has been designed.

The procedure is based on the determination of the parameters D and E of the relation $Err = D \cdot J + E$, using the grid encoder in order to record the position of the head during the execution of certain trajectories, and a simple model to predict the productivity in function of the Jerk.

This approach can be extended to any trajectory as long as the relationship between the error and the Jerk remains linear.

The experimental tests have been performed on two milling centres, one new (SPARK) and the other one in work (JOTECH). Several tool paths have been tested in order to determine the parameters D and E. These parameters are influenced from the axis in movement and from the velocity. The optimal value of Jerk depends from the ratio k_2/k_1, or the relative importance between the cost of time delay and cost of error. For the JOTECH, it has been determined experimentally.

Some numerical examples show that the cost reduction reaches 25%. The cost saving is greater when the segment is short and the sensitivity of the error than the Jerk (D) is significant.

The relation between the contouring accuracy and the Jerk can be determined through more complex models (more DOF) of the milling centre. The model should include the mechanical structure, and all the components have to be characterized through a calibration.

Another topic that will be covered is the determination of D through the encoder of the single axis of the machine.

In the future, other situations will be taken into account, i.e. if the feedrate changes during the tool path, due to small segments, feedrate scheduling or look ahead and for the tool paths made by several small linear segments, where each segment can have a different value of feedrate or a different imposed acceleration.

The objective function can be more complex, i.e. include the quality of the surface or the wear of the tool (even if these can be correlated with the overshoot).

This article has established a procedure for assigning a value to the Jerk, a parameter that will become increasingly important for high-speed machining of complex surfaces, especially for mechanical structures that are lightweight and not very stiff.

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